unit-1 [Nuclear physics]

Nuclear physics is the branch in which we discuss different properties of a nucleus and different phenomenon related to the nucleus. The size of nucleus is very small as compare to size of atom.

size of atom = 10^{5} x size of nucleus.

Nucleus contain most of the mass of an atom and the nucleus is form mainly by proton ~~or~~ and neutron.

Q. Define Nuclear force. Writedown different characteristics of Nuclear force.

→ The force between Nucleons (proton - proton), Neutron - Neutron, proton - neutron), which is responsible for the formation of neucleus is known as Nuclear force.

Nuclear force binds proton and neutrons to ~~form~~ formed a nucleus.

*** characteristics of Nuclear force:

i) Nuclear force is the strongest force in the nature.

ii) Nuclear force is charge independent.

iii) Nuclear force is a short range force, in acts within the nucleus. (10^{-14} to 10^{-15})

iv) Nuclear force is always attractive in nature.

v) Nuclear force is non-central force and non-conservative in nature, its depends on the velocity of the nucleons (proton - nutron) within the nucleus.

vi) Nuclearforce is spin dependent.

vii) Nuclear force has saturation property, that is if it increase number of nucleons the nuclear force per neucleon is remain constant for heavy atoms.

Q. Discuss about different types of ~~the~~ nucleus.

→ i) Isotops: The nucleus with same proton number ore atomic number but different mass number.

For example - ${}_{6}C^{12}$, ${}_{6}C^{14}$

ii) Isobars: The nuclei with different atomic no. and same mass number. For example - ${}_{6}C^{14}$, ${}_{7}N^{14}$

iii) **Isotones:** The nuclei with same neutron number but ~~prot~~ different proton number. For eg:- $_8O^{16}$, $_7N^{15}$

iv) **Mirror nuclei:** If the proton no. and ~~mass~~ neutron no. of two nuclei interchange with each other then the nuclei is called mirror nuclei.

For example: $_7N^{15}$, $_8O^{15}$

Q. Find the relation between radius of a nucleus and mass number.

→ Let, R is the radius of a nucleus of mass number ~~of~~ A.

The volume of nucleus is proportional to the number of nucleon. (proton - neutron).

$\Rightarrow \frac{4}{3}\pi R^3 \propto A$ [we consider the shape of nuclues as spherical]

$\Rightarrow R^3 \propto A$

$\Rightarrow R \propto A^{1/3}$

$\Rightarrow \boxed{R = R_o A^{1/3}}$ where $R_o = 1.4$ fm [1 fm $= 10^{-15}$ m] for light nucleus ($A < 30$)

$R_o = 1.3$ fm fore heavy nucleus ($A > 30$)

Q. Find the density of a ~~nuclus~~ nucleus.

→ The mass density of a nucleus $\rho = \frac{\text{mass}}{\text{volume}}$

$= \frac{M}{\frac{4}{3}\pi R^3}$

$= \frac{m_p \cdot \cancel{A}}{4/3 \pi R_o^3 \cdot \cancel{A}}$

$= \frac{3 m_p}{4\pi R_o^3}$

$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.4 \times 10^{-15})^3}$

$= \frac{5.01}{12.56 \times 2.744 \times 10^{-45}}$

$=$

$\simeq 10^{17}$ Kg/m^3

~~mass ≃ mar~~

mass proton ≃ mass neutron

$M =$ mass proton × mass number

$R =$ radius of the nucleus $= R_o A^{1/3}$

$m_p = 1.67 \times 10^{-27}$ Kg

$R_o = 1.4$ fm $= 1.4 \times 10^{-15}$ m.

The number density or number of nucleon per unit volume of the nucleus

$$= \frac{\text{Mass number}}{\text{Volume of nucleus}}$$

$$= \frac{A}{\frac{4}{3}\pi R^3}, \quad \Big| \; R = R_0 A^{1/3}$$

$$= \frac{\cancel{A}}{\frac{4}{3}\pi R_0^3 \cancel{A}} \qquad R_0 = 1.4\,fm.$$

$$= \frac{3}{4\pi R_0^3}$$

$$\simeq 10^{44} \text{ nucleon}/m^3$$

Note: periodically the density of a nucleus is constant throughout the nucleus but from experimental result we get the density of the nucleus is not constant, the density changes with the distance from the ~~centra~~ centre of nucleus.

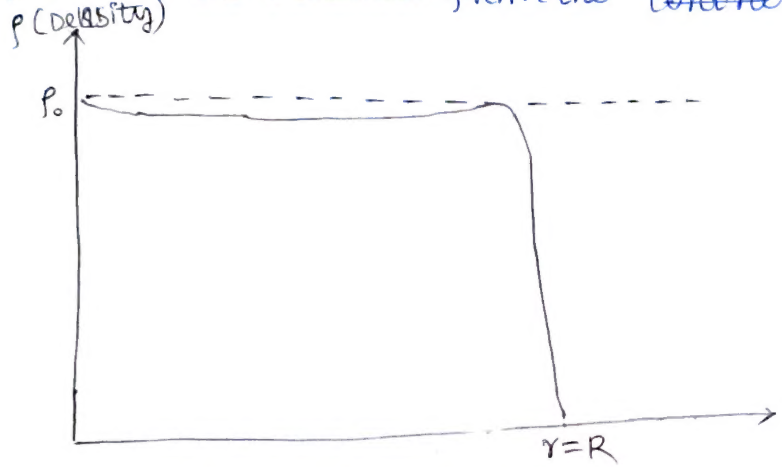


fig: Experimental graph for ρ vs r.

R = radius of nucleus.

Q. The radius of copper nuclues (Z = 29) is 6.2 fm. Find the radius of calcium nuclase (Z = 20).

→ Here R_1 = 6.2 fm. for cu.

mass number
cu (A) = 63
Ca (A) = 40

$$R_1 = R_0 A^{1/3}$$

$$R_1 = 6.2 \times 63^{1/3}$$

$$R_2 = R_0 \times 40^{1/3}$$

$$\frac{R_1}{R_2} = \frac{\cancel{R_0} \times 63^{1/3}}{\cancel{R_0}\, 40^{1/3}}$$

$$\text{or,} \quad \frac{6.2}{R_2} = 1.163$$

$$R_2 = 5.3\, fm.$$

Q. i) Define the term mass defect and binding energy of nucleus. Hence find an expression for binding energy.

ii) Draw a graph of binding energy per nucleon vs mass number A for light, medium, heavy nucleus. Hence discuss various factors of binding energy related to the graph. (2018)

→ i) When a nucleus is formed ~~fort~~ from its constituent particles (proton, neutron), it is observed that its mass is less than the sum of masses of all the constituent particle.

When Z no. of protons and N no. of neutrons combine to form a nucleus, sum of the mass (Δm) disappears because it gets convert to equal amount of energy. The mass that convert into equal amount of energy is known as mass defect, This equal amount of energy is called nuclear binding energy as this amount of energy combine proton and neutron ~~we~~ to form the nucleus.

Let, mass of proton = m_p and mass of nutron = m_n.
If the mass of the nucleus with Z number of protons and N no. of neutrons is ${}_ZM^A$ then mass defect is given by

$$\Delta m = Zm_p + (A-Z)m_n - {}_ZM^A$$

Binding energy, B.E $= \Delta mc^2$

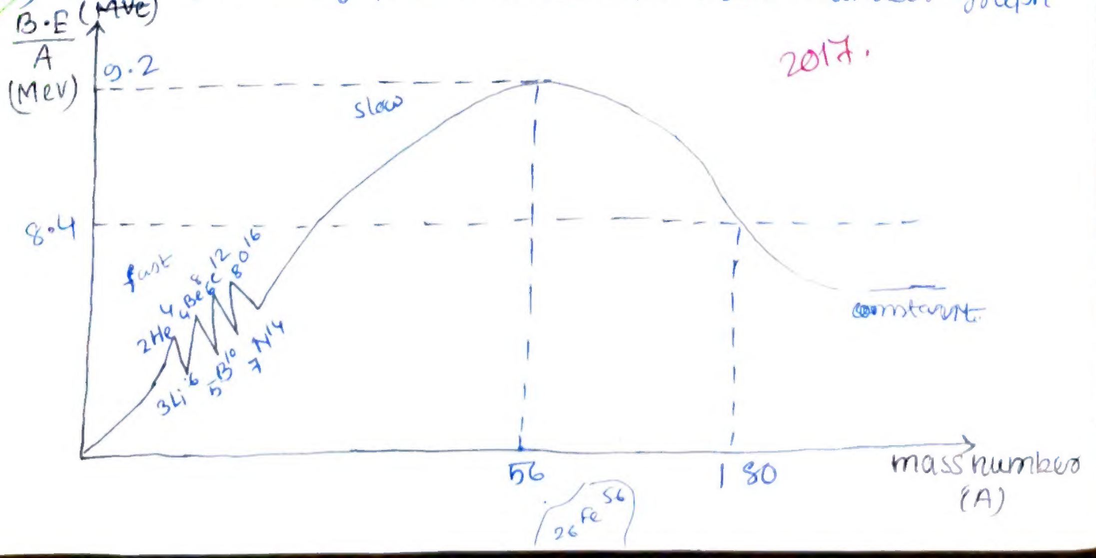
$$= \{ Zm_p + (A-Z)m_n - {}_ZM^A \} c^2$$

Binding energy per nucleon

$$\frac{B.E}{A} = \frac{\{ Zm_p + (A-Z)m_n - {}_ZM^A \} c^2}{A}$$

ii) Binding energy per nucleon vs mass number graph

2017.



Various factors related to this graph :-

i) For nuclei with mass number less than 20, there are some peaks in the ~~finding~~ binding energy curve corresponding to even even nuclei such as $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ etc.

This peaks are observed because proton and neutrons formed shell like structures within the nucleus same as electron, the nucleus with even-even number of proton neutron is more stable as compare to the nucleus with odd-odd number of proton-neutron.

ii) Binding energy per nucleon is very small for like nucleus and goes on increasing rapidly with increasing mass number and reaches a value 8.4 MeV for mass no. nearly 30. There after the rise of the curve is much slower, reaches a maximum value 9.2 MeV for mass no. nearly 56. if the mass number increased further the binding energy per nucleon ~~p~~ decreases slowly.

iii) For mass number greater 180 i.e for heavy nucleus the binding energy per nucleon is saturated to a value nearly 8 MeV.

iv) The variation of binding energy per nucleon for nucleuses within the mass number 30 to 180 varies ~~so~~ slowly as compared to light nuclei.

Q. Define separation energy. Why separation energy of proton is less than separation energy neutron.

→ The amount of energy required to separate a nucleon (proton - neutron) ~~for~~ from the nucleus is known as separation energy.

There are two types of separation energy —

i) proton separation energy.

ii) Neutron " " .

As the protons repels each other due to columbs force it is easier to separate a proton from the nuclues as compare to nutron. Thats why separation energy of proton is less than separation of energy neutron.

Q. Find the Binding energy per nucleon for copper given $m_n = 1.008665$ amu. $m_p = 1.007825$ amu.
mass of copper $M(cu^{63}) = 62.929599$ amu.

→ $Z = 29$

mass defect,
$\Delta m = 29 \times m_p + (63-29) m_n - M({}_{29}cu^{63})$

$\Delta m = 29.226925 + 34.29461 - 62.929599$

$\Delta m = 0.591936$ amu. $\boxed{1 \text{ amu.} = 931.2 \text{ MeV.}}$

Binding energy (B.E) $= \Delta m c^2$

$= 0.591936 \times \cancel{9 \times 10^{16}} \times 931.2$

$= \cancel{5.327424 \times 10^{16}}\ 551.21$ MeV.

Binding energy per cu nucleon,

$= \frac{B.E}{A}$

$= \frac{\cancel{5.327424 \times 10^{16}}\ 551.21}{63}$ MeV

$= \cancel{0.0846 \times 10^{16}}$

$= 8.7494$ MeV.

Q. i) write down Bethe-Weizsacker semi-empirical mass formula. Mention in different terms? 2018

ii) write down and explain different terms contributing in binding energy of nucleus.

OR, Explain the origin of different terms involved in semi-empirical mass formula.

→ (experimental + theoritical)

$\boxed{\text{empirical} \rightarrow \text{experimental}}$

→ i) Bethe weizsacker semi-empirical mass formula

we know that, from the defination of binding energy, the mass of nucleus can be expressed as

$${}_Z M^A = Z m_p + (A-Z) m_n - (B.E)/c^2$$

where ${}_Z M^A$ = mass of nucleus with atomic no. Z and mass no. A.

m_p = mass of proton.

m_n = " " Neutron.

B.E = Binding energy.

$E = mc^2$

$m = \frac{E}{c^2}$

Bloom: A flower (usually one on a plant that people admire for its ~~foto~~flower) প্রস্ফুটিত।

Syno: Freshness, flush.

Anto: Decay, withered state.

From Bethe weizscker mass formula the binding energy.

$$B.E = a_V A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_{as} \frac{(A-2Z)^2}{A}$$

$$(\pm a_p A^{-3/4} \text{ or zero}).$$

Here, $a_V A$ = volume energy term.
$-a_s A^{2/3}$ = surface " " .
$-a_c \frac{Z^2}{A^{1/3}}$ = colomb " " .
} theoritical

$-a_{as} \frac{(A-2Z)^2}{A}$ = Asymmetric energy term.
$\pm a_p A^{-3/4}$ or, zero = pairing energy term
} experimental.

ii) The different terms of semi-epirical mass formula for binding energy is given by,—

i) Volume energy term: we know that, nucleons (proton & neutron) attract each other by strong nuclear force.

If 'u' is the interaction energy betn two nucleons then contribution of each nucleon is $u/2$.

The density of nucleus is very high i.e the nucleons are very tightly ~~pad~~ packed inside the nucleus, the no. of nearest nucleon for each nucleon = 12.

So, interaction energy for each electron is $12 \times \frac{u}{2}$
$= 6u$.

If 'A' (mass number) is total number of nucleon, then total volume energy or interaction energy

$= 6uA$

$E_V = a_V A$, where a_V = constant.

ii) Surface energy term: All the ~~nucle~~ nucleons of ~~the~~ a nucleus is not present inside the volume, there is some nucleons at the surface of the nucleus. The nucleons that are present on the surface do not have symmetrical nuclear force.

nucleons of surface.

nucleons inside volume.

The nucleons that are present in the surface of the nucleus tries to back the stability of the nucleus, i.e this nucleons reduces the binding energy.

The no. of nucleons present on the surface is proportional to surface area of the nucleus.

Surface energy is $\propto$ surface area of nucleon

$$E_s \propto -4\pi R^2 \quad [R = \text{radius of nucleus}]$$

$$\text{or, } E_s \propto -4\pi r_0 A^{2/3} \quad [R = r_0 A^{1/3}]$$

$$\text{or, } E_s = a_s A^{2/3}; \quad a_s = \text{constant.}$$

iii) <u>Coloumb energy term</u>: The protons inside a nucleus repels each other due to colomb interaction force. The coloumb interaction force reduces the binding energy or sability of the nucleus.

If there are Z no. of proton inside the nucleus the no. of interactions $= \frac{Z(Z-1)}{2}$

The coloumb interaction energy each pair of proton

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r_{av}}; \quad r_{av} = \langle av \rangle \text{ distance bet}^n \text{ two proton.}$$

$$r_{av} \simeq R \text{ (Radius of nucleus)}$$

$$\simeq \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{R} \qquad R = r_0 A^{1/3}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r_0 A^{1/3}}.$$

The total coloumb interaction energy,

$$E_c = -\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r_0 A^{1/3}} \cdot \frac{Z(Z-1)}{2}$$

$$E_c = -a_c \frac{Z(Z-1)}{A^{1/3}}; \quad a_c = \text{constant.}$$

iv) <u>Assymmetric energy term</u>: From experimental result we get, the binding energy of a nucleus depends on the difference in number of neutron and proton,

Experimental data shows that;

$$E_{as} \propto (N-Z)^2$$

$$E_{as} \propto \frac{1}{A}$$

$\therefore\ E_{as} \propto -\frac{(N-Z)^2}{A}$; Mass no. $A = N+Z$
$N = A-Z$

$E_{as} = -a_{as}\frac{(A-2Z)^2}{A}$, a_{as} = constant.

v) pairing energy term: As like electrons proton & neutrons form there individual energy level shell inside the ~~or~~ nucleus.
Similar to electrons for even number of proton or neutron the nucleus is more stable as compared to odd number of proton or neutron.
from experimental data, the pairing energy term

$E_p = +a_p A^{-3/4}$ (even-even number of proton-neutron)
$= -a_p A^{-3/4}$ (odd-odd " " " ")
(unstable)
$= 0$ (even-odd no. of proton-neutron).

Q. By using semi-epirical mass formula find the expression for atomic no. of most stable isobar.

→ From semi-epirical mass formula the binding energy of nucleus with atomic number 'Z' and mass number 'A'

$$B.E = a_v A - a_s A^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_{as}\frac{(A-2Z)^2}{A}$$
$$(\pm a_p A^{-3/4}, 0).$$

For most stable isobar,

$$\left[\frac{d}{dZ}(B.E)\right]_{A=\text{constant.}} = 0$$

or, $-\frac{a_c \cancel{(Z-1)}}{A^{1/3}}(2Z-1) - a_{as}\cdot\frac{1}{A}\cdot 2(A-2Z)\cdot(-2) = 0$

or, $\frac{a_c}{A^{1/3}}\cdot(2Z-1) = \frac{4a_{as}(A-2Z)}{A}$

or, $\frac{2a_c Z}{A^{1/3}} - \frac{a_c}{A^{1/3}} = 4a_{as} - \frac{8a_{as}Z}{A}$

or, $Z\left(\frac{2a_c}{A^{1/3}} + \frac{8a_{as}}{A}\right) = 4a_{as} + \frac{a_c}{A^{1/3}}$

or, $\frac{2Z}{A^{1/3}}\left(a_c + \frac{4a_{as}}{A^{2/3}}\right) = \frac{A^{1/3}4a_{as} + a_c}{A^{1/3}}$

or, $2Z\frac{a_c A^{2/3} + 4a_{as}}{A^{2/3}} = 4a_{as}A^{1/3} + a_c$

or, $\boxed{Z = \frac{4a_{as}A + a_c A^{2/3}}{2a_c A^{2/3} + 8a_{as}}}$

This is the expression for most ~~sta~~ atomic no. of most stable isobar.

Q. Find the most stable isobar for atomic no. for mass number 12. Given $a_c = 0.714$ MeV & $a_{as} = 23.21$ MeV.

$$\rightarrow \quad Z = \frac{4 \times 23.21 \times 12 + 0.714 \times 12^{2/3}}{2 \times 0.714 \times 12^{2/3} + 8 \times 23.21} \rightarrow 5.241$$

$$= \frac{1114.08 + 3.742}{7.484 + 185.68}$$

$$= \frac{1117.822}{193.164} = 5.787$$

Q. By using semi-empirical mass formula find the binding energy per nucleon for the following nucleus.

i) $_{26}Fe^{56}$ ii) $_{92}U^{235}$

Given $a_v = 15.85$ MeV, $a_s = 18.34$ MeV.

$a_c = 0.714$ MeV, $a_{as} = 23.21$ MeV.

$a_p = 12$ MeV.

$\rightarrow$ i) $_{26}Fe^{56}$; proton $= 26$ $\quad$ $A = 56$,

Neutron $= 30$

From semi-empirical mass formula,

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{as} \frac{(A-2Z)^2}{A} + a_p A^{-3/4}$$

$$= 15.85 \times 56 - 18.34 \times 56^{2/3} - 0.714 \times \frac{26(26-1)}{56^{1/3}} - 23.21 \times \frac{(56 - 2 \times 26)^2}{56} + 12 \times 56^{-3/4} \text{ MeV}$$

$= 491.80$ MeV. or 491.67.

B.E per nucleon energy

$$= \frac{491.806}{56}$$

$= 8.778$ MeV.

(ii) $_{92}U^{235}$, $A = 235$.

proton = 92
neutron = 143

From semi-empirical mass formula,

$$B.E = a_V A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{as} \frac{(A-2Z)^2}{A} + a_p A^{-3/4}$$

$$= 15.85 \times 235 - 18.34 \times (235)^{2/3} - 0.714 \times \frac{92 \times 91}{(235)^{1/3}} - 23.21 \times \frac{(235 - 2\times 92)^2}{235} + 12 \times (235)^{-3/4}$$

$= 1800.99$ MeV

B.E per nucleon energy,

$= \frac{1800.99}{235}$ MeV.

$= 7.664$ MeV

Q. Compare between stability of following nuclei $_6C^{12}$, $_6C^{13}$, $_6C^{14}$ by using semi-empirical mass formula.

bin B.E large stable.

Given..... $a_V = 15.85$ MeV. $a_s = 18.34$ MeV. $a_c = 0.714$ MeV. $a_{as} = 23.21$ MeV. $a_p = 12$ MeV.

For, $_6C^{12}$ proton = 6 Nutron = 6 $A = 12$

From semi-empirical mass formula,

$$B.E = a_V A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{as} \frac{(A-2Z)^2}{A} + a_p A^{-3/4}$$

$$= 15.85 \times 12 - 18.34 \times (12)^{2/3} - 0.714 \times \frac{6 \times 5}{(12)^{1/3}} - 23.21 \times \frac{(12-12)^2}{12} + 12 \times (12)^{-3/4}$$

$= 86.576$ MeV.

For $_6C^{13}$ proton no. = 6 $\quad A = 13$

neutron no. = 7

From semi empirical mass formula,

$$B.E = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{as} \frac{(A-2Z)^2}{A} + 0$$

$$= 15.85 \times 13 - 18.34 \times (13)^{2/3} - 0.714 \times \frac{6(6-1)}{(13)^{1/3}} - 23.21 \times \frac{(13-12)^2}{13}$$

$$= 93.76$$

For $_6C^{14}$ proton no. = 6 $\quad A = 14$

Neutron = 8

From semi epirical mass formula,

$$B.E = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{as} \frac{(A-2Z)^2}{A} + a_p A^{-3/4}$$

$$= 15.85 \times 14 - 18.34 \times (14)^{2/3} - 0.714 \times \frac{6 \times 5}{(14)^{1/3}} - 23.21 \times \frac{(14-12)^2}{14} + 12 \times (14)^{-3/4}$$

$$= 101.50$$

Q. State the conservation laws that are obey during nuclear reactions? (2018) 2014

→ Nuclear reactions are governed by the following conservation laws.

i) conservation of charge: The charge of the reactants and the products are equal, i.e the total charge before and after the reaction is conserved.

for example: $_7N^{14} + _2He^4 \longrightarrow _8O^{17} + _1H^1 + \text{energy}$.

charge before reaction = 9e

" after " = 9e

ii) conservation of mass number: The total mass number of the reactants before the reaction is must be equal to the mass no. of the products after the reaction.

In the above example, mass no. of the reactants = 14 + 4 = 18

mass no. of the product = 17 + 1 = 18

iii) conservation of mass energy:

iv) conservation of linear momentum & angular momentum.

v) conservation of spin.

Q. Determine the unknown particle in the following Nuclear reaction, $_{20}Ca^{42}(_3Li^6, x)\ _{21}Sc^{45}$

→ The following nuclear reaction,

$$_{20}Ca^{42} + _3Li^6 \longrightarrow _{21}Sc^{45} + x_2^3$$

$\therefore\ x \longrightarrow\ _2He^3$

Q. i) Define Q value of a nuclear reaction

ii) Express Q value in terms of kinetic energy, nuclear mass, binding energy of reactants and products.

iii) write down the significance of Q value.

→ i) Q value is defined as the total energy released or absorbed in a nuclear reaction.

We consider a nuclear reaction, X (x, y) Y, in which a fast moving particle x with kinetic energy K_x is incident on a ~~nucleu~~ nucleus X (supposed to be at rest).

The products of the nuclear reactions y & Y, with kinetic energy K_y & K_Y

The Q value of the reaction is given by

$$Q = \text{K.E of product} - \text{K.E of reactant}$$

$$Q = K_y + K_Y - K_x \longrightarrow (i)$$

according to conservation of mass energy

$$m_X c^2 + m_x c^2 + K_x = m_y c^2 + K_y + m_Y c^2 + K_Y$$

or, $(K_y + K_Y) - K_x = \{(m_X + m_x) - (m_y + m_Y)\}c^2$

or, $Q = \{\text{mass of Reactant} - \text{mass of product}\}c^2 \longrightarrow (ii)$

Let, the nuclear reaction

$${}_{Z_1}X^{A_1} + {}_{Z_2}x^{A_2} \longrightarrow {}_{Z_3}y^{A_3} + {}_{Z_4}Y^{A_4}$$

By conservation of charge and mass number,

$$Z_1 + Z_2 = Z_3 + Z_4$$

$$A_1 + A_2 = A_3 + A_4$$

Total B.E of Reactant,

$$= \{Z_1 m_p + (A_1 - Z_1)m_n - m_X\}c^2 + \{Z_2 m_p + (A_2 - Z_2)m_n - m_x\}c^2$$

m_p = mass of proton,
m_n = " " Nutron.

Total B.E of product,

$$= \{Z_3 m_p + (A_3 - Z_3)m_n - m_y\}c^2 + \{Z_4 m_p + (A_4 - Z_4)m_n - m_Y\}c^2$$

Now, B.E of product $-$ B.E of reactant

$$= \{Z_3 m_p + (A_3 - Z_3)m_n - m_y\}c^2 + \{Z_4 m_p + (A_4 - Z_4)m_n - m_Y\}c^2 - \{Z_1 m_p + (A_1 - Z_1)m_n - m_X\}c^2 - \{Z_2 m_p + (A_2 - Z_2)m_n - m_x\}c^2$$

$$= (Z_3 + Z_4 - Z_1 - Z_2)m_p c^2 + (A_3 - Z_3 + A_4 - Z_4 - A_1 + Z_1 - A_2 + Z_2)m_n c^2 - m_y c^2 - m_Y c^2 + m_X c^2 + m_x c^2$$

$$= (m_x + m_X)c^2 - (m_y + m_Y)c^2 + 0 \times m_p c^2 + 0 \times m_n c^2$$

$= (m_x + M_X)c^2 - (m_y + M_Y)c^2$

$= Q$ value.

$\therefore \boxed{Q \text{ value} = B.E \text{ of product} - B.E \text{ of reactant.}}$

iii) Significance of Q values

The Q value of a nuclear reaction denotes the energy released or absorbed in such a nuclear reaction. Also this gives a physical varification of conversion of mass into equivalent amount of energy and ~~vise~~ viz-versa.

Q i) Define exothermic & endothermic nuclear reaction

ii) Define Threshold kinetic energy of endothermic reaction.

→ i) If the Q value is +ve in a nuclear reaction i.e energy is released in a nuclear reaction, then it is known as exothermic Nuclear reaction. In exothermic nuclear reaction a fraction of mass of the reactant convert into kinetic energy by the law of mass energy conversion.

If the Q-value is -ve in a nuclear reaction i.e total energy is absorbed in a nuclear reaction, then it is called endothermic nuclear reaction. In this case a fraction of kinetic energy of the reactant converts into mass of the product.

ii) The minimum kinetic energy of the reactant or the projectile to start an endothermic reaction is known as threshold K.E of endothermic reaction.

$$\underset{\text{projectile}}{x} + \underset{\text{Rest}}{X} \xrightarrow{Q<0} y + Y$$

The threshold kinetic energy for this reaction,

$$K_{Th} = -Q\left(1 + \frac{m_x}{M_X}\right)$$

Threshold energy, $\boxed{E_{Th} = m_x c^2 + K_{Th}}$

Q. Calculate the Q value of the following reaction

$${}_1H^2 + {}_{29}Cu^{63} \longrightarrow {}_0n^1 + {}_{30}Zn^{64}$$

if ${}_1H^2$ of K.E = 12 MeV incident on target ${}_{29}Cu^{63}$ at rest, a neutron is observed with K.E 16.85 MeV. Find the K.E of ${}_{30}Zn^{64}$?

Given $m({}_1H^2) = 2.014102$ amu,
$m(n) = 1.008665$ amu, $m({}_{29}Cu^{63}) = 62.929599$ amu
$m({}_{30}Zn^{64}) = 63.929142$ amu.

$\rightarrow$ $Q = \{\text{mass reactant} - \text{mass product}\}c^2$

$= [\{m({}_1H^2) + m({}_{29}Cu^{63})\} - \{m(n) + m({}_{30}Zn^{64})\}]c^2$

$= \{(2.014102 + 62.929599) - (1.008665 + 63.929142)\}c^2$

$= (0.005894 \text{ amu})c^2$

1amu $\equiv$ 931.2 MeV

$= 0.005894 \times 931.2$ MeV

$= 5.4885$ MeV.

$\therefore$ Q value $= 5.49$ MeV.

Cu = rest
= K.E = 0

Again, Q value = K.E of product − K.E Reactant

$5.49 \text{ MeV} = 16.85 + x - 12$

or, $x = 0.64$ MeV.

$\therefore$ K.E of ${}_{30}Zn^{64}$ is 0.64 MeV.

Q. Calculate the threshold kinetic energy for the following nuclear reactions if proton is incident on ${}_1H^3$ (tritium) at rest.

$$p + {}_1H^3 \longrightarrow {}_1H^2 + {}_1H^2$$

Given, $m(p) = 1.007825$ amu, $m({}_1H^3) = 3.016049$ amu
(m_x) (m_X)
$m({}_1H^2) = 2.014102$ amu.

$\rightarrow$ $Q = \{\text{mass reactant} - \text{mass product}\}c^2$

$= \{(1.007825 + 3.016049) - (2.014102 + 2.014102)\}c^2$

$= -0.00433 \times c^2$ amu

$= -0.00433 \times 931.2$ MeV

$= -4.032096$ MeV.

$$K_{Th} = -Q\left(1 + \frac{m_x}{m_X}\right)$$

$= 4.032056 \left(1 + \frac{1.007825}{3.016049}\right)$

$= 4.032056 \times 1.33412$

$= 5.38$

Q. Define radio activity. show that, for a radio active sample, no. of radioactive nucleus at time t $N = N_0 e^{-\lambda t}$ where N_0 = number of nucleus at $t = 0$, λ = Decay constant.

→ Some heavy nuclei emit particles continuously to become stable. this process of continuous emission is known as radioactivity and the materials are known as radio active material. Heavy nuclei emit α, β, γ particle respectively to become stable.

Let, N_0 is the number of nucleus at time $t = 0$ in the radio active sample.

The rate of decay ~~of~~ or disintegration is proportional to the number of radio active nuclei at that time in the sample.

$$\therefore \frac{dN}{dt} \propto -N \; ; \quad N = \text{number of radioactive nuclei at time } t.$$

$$\text{or, } \frac{dN}{dt} = -\lambda N \; , \quad \lambda = \text{decay constant.}$$

Integrating both side with proper limit

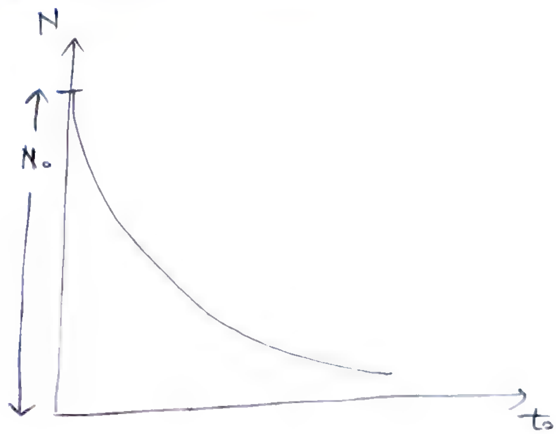
$$\int_{N_0}^{N} \frac{dN}{dt} = -\lambda \int_0^t dt$$

$$\text{or, } [\ln N]_{N_0}^{N} = -\lambda t$$

$$\text{or, } \ln \frac{N}{N_0} = -\lambda t$$

$$\text{or, } \frac{N}{N_0} = e^{-\lambda t}$$

$$\text{or, } \boxed{N = N_0 e^{-\lambda t}}$$



Q. Define half life & mean life for a radio active sample. Hence find the relation between them.

→ The time at which the number of radio active nuclei in the sample is become half of the no. of the nuclei at $t = 0$, is known as half life.

Let, $t = t_{1/2}$, $N = N_0/2$

$$\therefore \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\text{or, } \lambda t_{1/2} = \ln 2$$

$$\text{or, } t_{1/2} = \frac{\ln 2}{\lambda}$$

$$\text{or, } \boxed{t_{1/2} = \frac{0.693}{\lambda}}$$

The time at which the no. of radio active nuclei of the sample is become $1/e$ times of radio active nuclei at $t=0$, is known as mean life.

let, $t = \tau$, $N = \frac{N_0}{e}$

$$\therefore \frac{N_0}{e} = N_0 e^{-\lambda\tau}$$

or, $e^{-1} = e^{-\lambda\tau}$

or, $\lambda\tau = 1$

or, $\tau = \frac{1}{\lambda}$

or, $t_{1/2} = \frac{0.693}{\lambda}$

$$t_{1/2} = 0.693 \times \tau$$

$$\tau > t_{1/2}$$

Q. Define activity of a radio active sample. Find an expression for activity at time t.

→ The disintegration rate for a radio active sample is known as activity.

Activity, $A = \left|\frac{dN}{dt}\right| = \lambda N$

initial activity ($t=0$), $A_0 = \lambda N_0$

we know that,

$$N = N_0 e^{-\lambda t}$$

$$\lambda N = \lambda N_0 e^{-\lambda t}$$

or, $A = A_0 e^{-\lambda t}$

This is the expression for activity at time t.

unit of activity is DPS or Bq

(DPS = Disintegration per sec.)

Q. State the law of Radio active decay.

→ $_Z X^A \xrightarrow{-\alpha} {}_{Z-2}Y^{A-4}$

$_Z X^A \xrightarrow{-\beta^-} {}_{Z+1}Y^{A}$

$_Z X^A \xrightarrow{-\beta^+} {}_{Z-1}Y^{A}$

When a nucleus emits α particle its ~~atom~~ atomic no. decreases by two and mass no. decreases by 4. When a nucleus emits β^- particle its mass no. remain unchange but the atomic number increased by one, if β^+ particle emited from the nucleus the mass number remains unchanged but the atomic no. decreased by one.

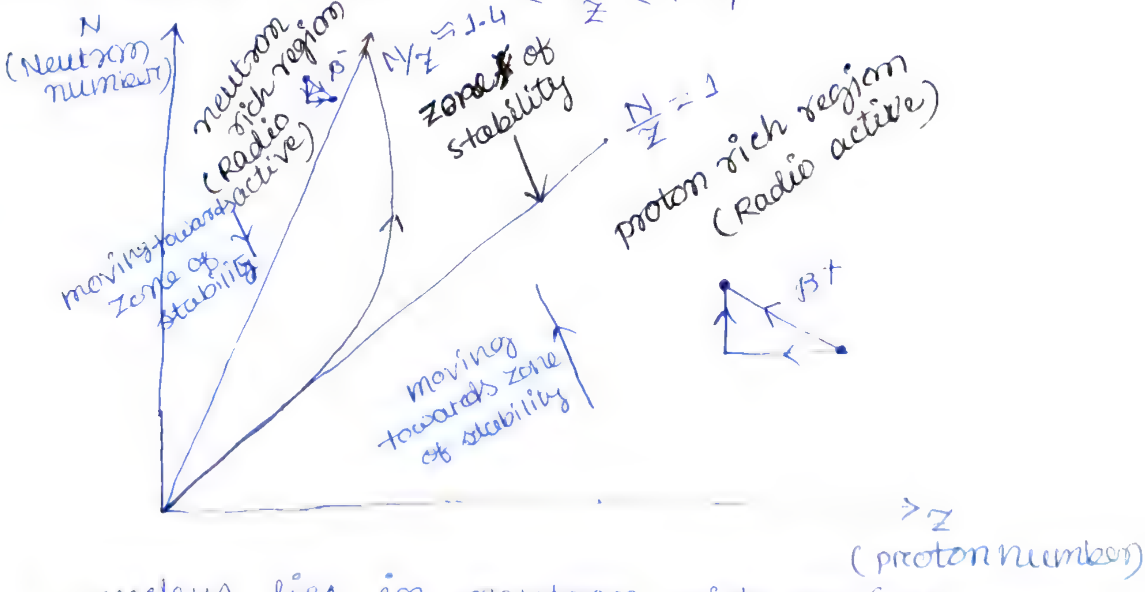
Q. Explain the stability of nucleus on neutron to proton number ratio.

→ The stability of nucleus means that it can not decay spontinuously like radio active materieal. the stability of nucleus can be explain ~~into~~ terms of neutron to proton number reatio.

for light stable nucleus $\frac{N}{Z} = 1$; N = neutron number, Z = proton "

for heavy stable nucleus $\frac{N}{Z} \simeq 1.4$

So, for stable nucleus $1 \leq \frac{N}{Z} \leq 1.4$



If a nucleus lies in neutron reich region then one neutron converted into proton by emission of β^- particle.

$n \longrightarrow p + e^-$ (β^- particle)

If nucleus lies in proton rich region then one proton is convert into neutron

$p \longrightarrow n + e^+$ (β^+ particle or positron)

Q i) Define successive disintegration. Explain the basic theory of successive disintegration.

ii) In successive disintegration $P \longrightarrow Q \longrightarrow R$, show that time during which the daughter nucleus Q attains a max^n no. is given by, (2017)

$t_m = \frac{1}{\lambda_1 - \lambda_2} \cdot \ln \frac{\lambda_1}{\lambda_2}$ where λ_1 is decay const for $P \rightarrow Q$

λ_2 is decay const for $Q \rightarrow R$

→ Naturally Radio active materials disintegrate into other Radio active dauther nucleus, The daughter nucleus further disintegrated into other nucleus until a stable product is form. This type of disintegration is known as successive disintegration.

$^{235}_{92}U \xrightarrow{\alpha} {}^{231}_{90}Th \xrightarrow{\alpha} {}^{227}_{88}Lu \rightarrow \cdots \rightarrow {}_{82}Pb$

we consider a basic successive disintegration

~~$P \rightarrow Q \rightarrow R$~~

	P	Q	R
at $t=0$,	N_{P0}	0	0
at $t=t$;	N_P	N_Q	N_R

Let, at any time t the number of nucleus of P, Q, R are N_P, N_Q, N_R respectively and decay constant are λ_1 and λ_2

For $P \rightarrow Q$ disintegration; the rate of decay.

$$\frac{dN_P}{dt} = -\lambda_1 N_P$$; λ_1 = disintegration consts for $P \rightarrow Q$.

Integrating we get,

$$N_P = N_{P0}\, e^{-\lambda_1 t} \longrightarrow (i)$$

N_{P0} = No. of nuclei of P at the beginning.

Rate of disintegration of Q

$= -\lambda_2 N_Q$

Rate At the same time the rate of production of Q nuclei is equal to the rate of decay of P nuclei.

So, Rate of change of Q,

$$\frac{dN_Q}{dt} = -\lambda_2 N_Q + \lambda_1 N_P$$

or, $$\frac{dN_Q}{dt} = -\lambda_2 N_Q + \lambda_1 N_{P0} e^{-\lambda_1 t}$$

or, $$\frac{dN_Q}{dt} + \lambda_2 N_Q = \lambda_1 N_{P0} e^{-\lambda_1 t} \longrightarrow (ii)$$

Here I.F $= e^{\int \lambda_2 dt}$

$= e^{\lambda_2 t}$

Multiplying both side of equation (ii) by I.F we get,

$$d(N_Q \cdot e^{\lambda_2 t}) = \lambda_1 N_{P0} e^{-\lambda_1 t} e^{+\lambda_2 t} dt$$

or, $$d(N_Q e^{\lambda_2 t}) = \lambda_1 N_{P0} e^{-(\lambda_1 - \lambda_2)t} dt$$

Integrating both side

$$N_Q e^{\lambda_2 t} = \frac{\lambda_1 N_{P0}}{(\lambda_2 - \lambda_1)} \cdot e^{(\lambda_2 - \lambda_1)t} + c \text{ (integration const.)}.$$

at $t = 0$, $N_Q = 0$;

$$0 = \frac{\lambda_1 N_{P0}}{(\lambda_2 - \lambda_1)} + c$$

or, $$c = \frac{\lambda_1 N_{P0}}{(\lambda_2 - \lambda_1)}$$

Now, $\therefore$ $$N_Q e^{\lambda_2 t} = \frac{\lambda_1 N_{P0}}{(\lambda_2 - \lambda_1)} \left\{ e^{(\lambda_2 - \lambda_1)t} - 1 \right\}$$

or, * $$\boxed{N_Q = \frac{\lambda_1 N_{P0}}{(\lambda_2 - \lambda_1)} \left\{ e^{-\lambda_1 t} - e^{-\lambda_2 t} \right\}} \longrightarrow (iii)$$

Rate of production of R

$$\frac{dN_R}{dt} = \lambda_2 N_Q$$

$$dN_R = \frac{\lambda_1 \lambda_2 N_{P0}}{(\lambda_2 - \lambda_1)} \left\{ e^{-\lambda_1 t} - e^{-\lambda_2 t} \right\} dt$$

Integrating both side,

$$N_R = \frac{\lambda_1 \lambda_2 N_{P0}}{(\lambda_2 - \lambda_1)} \cdot \left\{ \frac{e^{-\lambda_1 t}}{-\lambda_1} + \frac{e^{-\lambda_2 t}}{\lambda_2} \right\} + c_1$$

At $t=0$, $N_R = 0$

$$0 = \frac{\lambda_1\lambda_2}{(\lambda_2-\lambda_1)} N_{P0} \left\{ -\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right\} + C_1$$

$$0 = \frac{\lambda_1\cancel{\lambda_2}}{(\lambda_2-\lambda_1)} N_{P0} \frac{\lambda_1-\lambda_2}{\lambda_1\cancel{\lambda_2}} + C_1$$

or, $C_1 = N_{P0}$

$$\therefore \boxed{N_R = \frac{\lambda_1\lambda_2 N_{P0}}{(\lambda_2-\lambda_1)} \left\{ \frac{e^{-\lambda_1 t}}{-\lambda_1} + \frac{e^{-\lambda_2 t}}{\lambda_2} \right\} + N_{P0}}$$

ii) From equation (iii)

$$N_Q = \frac{\lambda_1}{(\lambda_2-\lambda_1)} N_{P0} \left\{ e^{-\lambda_1 t} - e^{-\lambda_2 t} \right\}$$

for maximum Daughter nuclei Q;

$$\frac{dN_Q}{dt} = 0$$

or, $\frac{\lambda_1 N_{P0}}{(\lambda_2-\lambda_1)} \cdot \left\{ -\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} \right\} = 0$

or, $-\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} = 0$

or, $\lambda_2 e^{-\lambda_2 t} = \lambda_1 e^{-\lambda_1 t}$

or, $\frac{\lambda_2}{\lambda_1} = e^{(\lambda_2-\lambda_1)t}$

or, $(\lambda_2-\lambda_1)t = \ln\frac{\lambda_2}{\lambda_1}$

or, $t = \frac{1}{(\lambda_2-\lambda_1)} \cdot \ln\frac{\lambda_2}{\lambda_1}$

at time $t_m = \frac{1}{(\lambda_2-\lambda_1)} \ln\frac{\lambda_2}{\lambda_1}$

The no. of ~~bottom~~ daughter nucleus Q will be maximum.

*** Q. Explain transient equilibrium and secular equilibrium of successive disintegration.

→ We consider a successive disintegration

$$P \rightarrow Q \rightarrow R$$

	P	Q	R
at $t=0$	N_{P0}	0	0
at $t=t$	N_P	N_Q	N_R

Let, at time t the number of nuclei of P, Q, R is respectively N_P, N_Q, N_R and the disintegration constant λ_1, λ_2 respectively.

At time t, the number of nuclei of Q is

$$N_Q = \frac{\lambda_1 N_{P0}}{(\lambda_2-\lambda_1)} \left\{ e^{-\lambda_1 t} - e^{-\lambda_2 t} \right\} \longrightarrow (i)$$

when $\tau_P > \tau_Q$ i.e. $\lambda_2 > \lambda_1$ and $\left(\tau = \frac{1}{\lambda}\right)$

$t >> \tau_Q$ $(\lambda_2 >> t)$

$t >> \tau_Q$

$t >> \frac{1}{\lambda_2}$

$\lambda_2 >> t$

$e^{-\lambda_2 t} = \frac{1}{e^{\lambda_2 t}}$

From equation (i)

$$N_Q = \frac{\lambda_1 N_{P0}}{(\lambda_2-\lambda_1)} \cdot e^{-\lambda_1 t}$$

$$N_Q = \frac{\lambda_1}{\lambda_2-\lambda_1} N_P \; ; \text{ as } N_P = N_{P0}\, e^{-\lambda_1 t}$$

$$\frac{N_Q}{N_P} = \frac{\lambda_1}{\lambda_2-\lambda_1}$$

$$\text{or, } \frac{N_Q}{N_P} = \text{constant}$$

In such case both P & Q disintegrate while ratio of N_P and N_Q is remain constant. This type of equilibrium is known as trasient equalibrium.

Secular equilibrium: when $\tau_P >> \tau_Q$ that is $\lambda_2 >> \lambda_1$ and $t >> \tau_Q$ $(\lambda_2 >> t)$ then from eqn. (i) we get,

$$N_Q = \frac{\lambda_1 N_{P0}}{\lambda_2} e^{-\lambda_1 t} \; ; \text{ As } (\lambda_2 - \lambda_1) \simeq \lambda_2 \text{ and } \lambda_2 >> 1$$

$e^{-\lambda_2 t}$ is ~~rep.~~ neglected as $\lambda_2 >> t$.

$$N_Q = \frac{\lambda_1}{\lambda_2} N_P$$

$$\text{or, } \boxed{\lambda_2 N_Q = \lambda_1 N_P}$$

$N_P = N_{P0}\, e^{-\lambda_1 t}$

This shows that at equilibrium, the rate of decay of any radio active product is just equal to the rate of production from the previous member of successive disintegration. This type of equilibrium is known as Secular equilibrium.

$$\underbrace{\lambda_2 N_Q}_{\text{rate of disintegration of } Q} = \lambda_1 N_P \rightarrow \text{rate of disint. of } P$$

Q. i) Define packing fraction of a nucleus.
ii) How the packing fraction is related to with the binding energy of a nucleus.
iii) How does packing fraction is vary with mass no. of nucleus. (2018)

→ i) Packing fraction: The standard of mass used for comparison of masses is mass of $_6C^{12}$. The mass of $_6C^{12}$ is 12 a.m.u (atomic mass unit). If we measure the masses of other nuclei by mass spectroscopy, the masses seem to be very close to whole number but still there is a deviation from whole number. This deviation from whole number was expressed as packing fraction.

$$\text{packing fraction} = \frac{\text{Atomic mass of nucleus}(_ZM^A) - \text{Mass number}(A)}{\text{mass number}(A)}$$

$$\boxed{f = \frac{_ZM^A - A}{A}}$$

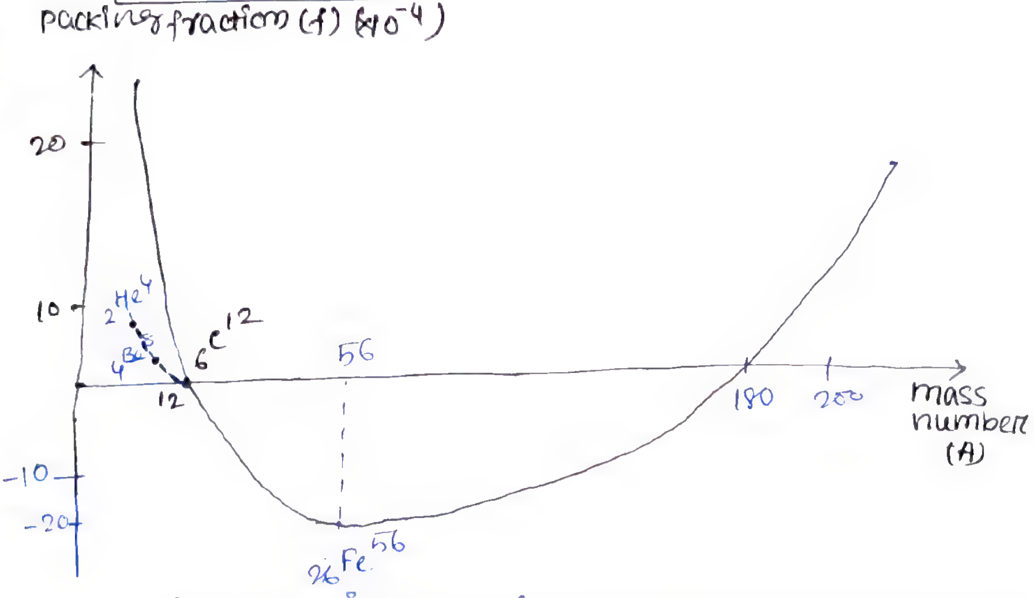


fig: packing fraction vs mass number graph

From the packing fraction vs mass number graph we find that –

i) For light nuclei (mass no. < 12), the packing fraction is maximum and positive, that indicating they are less stable.

ii) With the increasing mass number the packing fraction goes on decreasing till it become minimum for mass number 56.

iii) After mass number 56 the packing fraction start increasing ~~a~~gain with mass number but negative upto mass number 180.

iv) The value of packing fraction is positive after mass number 180 which indicate unstable radio active nuclei.

v) The packing fraction vs mass number graph is just opposite of binding energy per nucleon vs mass number graph, that is nuclei with high binding energy has negative packing fraction and nuclei with low binding energy has +ve packing fraction.

$\boxed{\alpha\text{- Disintegration}}$

Q. Show that, in α disintegration maximum kinetic energy is carry out by α-particle.

OR, show that, kinetic energy of α-particle emitted from a parent nuclei with mass number A is given by

$$K.E = \frac{A-4}{A} \times Q \; ; \; Q = Q\text{-value of the reaction.}$$

→ We consider, a parent nuclei ${}_Z X^A$, emitted a α-particle from rest.

$${}_Z X^A \longrightarrow {}_{Z-2}Y^{A-4} + {}_2\alpha^4$$

Let, the rest mass velocity of ~~total~~ ~~nucle~~ daughter nuclei Y is m_Y and velocity V_Y, mass of α-particle is m_α and velocity V_α

Using conservation of linear momentum

$$0 = m_Y V_Y + m_\alpha V_\alpha$$

$$\text{or, } m_Y V_Y = -m_\alpha V_\alpha \longrightarrow (i)$$

the Q value of the nuclear reaction

$$Q = \text{K.E of product} - \text{K.E of reactant}$$

$$Q = \frac{1}{2} m_\alpha V_\alpha^2 + \frac{1}{2} m_Y V_Y^2 - 0$$

$$= \frac{1}{2} m_\alpha V_\alpha^2 + \frac{1}{2} m_Y \cdot \frac{m_\alpha^2 V_\alpha^2}{m_Y^2} \quad [\text{using (i)}]$$

$$Q = \frac{1}{2} m_\alpha V_\alpha^2 \left\{ 1 + \frac{m_\alpha}{m_Y} \right\} \longrightarrow (ii)$$

We know that, mass of nucleus is proportional to mass number

$m_\alpha = 4$

$$\frac{m_\alpha}{m_Y} = \frac{4}{A-4}$$

From eqn. (ii)

$$Q = \frac{1}{2} m_\alpha v_\alpha^2 \cdot \left\{1 + \frac{4}{A-4}\right\}$$

$$= \frac{1}{2} m_\alpha v_\alpha^2 \cdot \frac{A}{A-4}$$

or, $\frac{1}{2} m_\alpha v_\alpha^2 = \left(\frac{A-4}{A}\right) \cdot Q$

$\therefore$ $\boxed{\text{K.E of } \alpha \text{ particle} = Q \times \frac{(A-4)}{A}}$

Q value = amount of energy release in the α disintegration

K.E of daughter nuclei, $= Q - Q\left(\frac{A-4}{A}\right)$

$$= \frac{4Q}{A}$$

[Q] An α-particle is emited from $^{64}_{30}Zn$, the Q value of this disintegration is 26.8 MeV. Find the kinetic energy of α particle and daughter nuclei.

→ $A = 64$, $Q = 26.8$ MeV.

K.E of α particle $= 26.8 \cdot \frac{60}{64}$

$= 25.125$ MeV

K.E of daughter nuclei, $= \cancel{26.8} \; \frac{4 \times 26.8}{\cancel{64}\, 16}$

$= 1.675$ MeV.

[Q] Define range of α-particle. State the factors on which range of α-particle depends.

→ Range of α-particle: The distance through which an α-particle travels in a specific material before stoping to ionise it, is known as range of α-particle.

Range of α-particle sharply defined ionisation path length. The Range of α-particle is highest in gaseous medium, less in liquid medium and least in solid medium due to more and more density of the medium.

In gaseous medium the range of α-particle depends on —

i) The initial kinetic energy of α-particle : For a specific medium the range of α-particle increases, with the increase in intial kinetic energy.

ii) Ionisation potential of the gaseous medium : The range of α-particle is inverselly proportional to ionization potential of the gas i.e the i with the increasing ionisation potential range of α-particle decreases

iii) pressure & temperature of gas : with increase of pressure the range of α-particle decreases, with increase in temperature, range of α particle decreases.

Q. What is Geiger-Nuttal law related to range of α-particle. Represents Geiger Nuttal law using a graph.

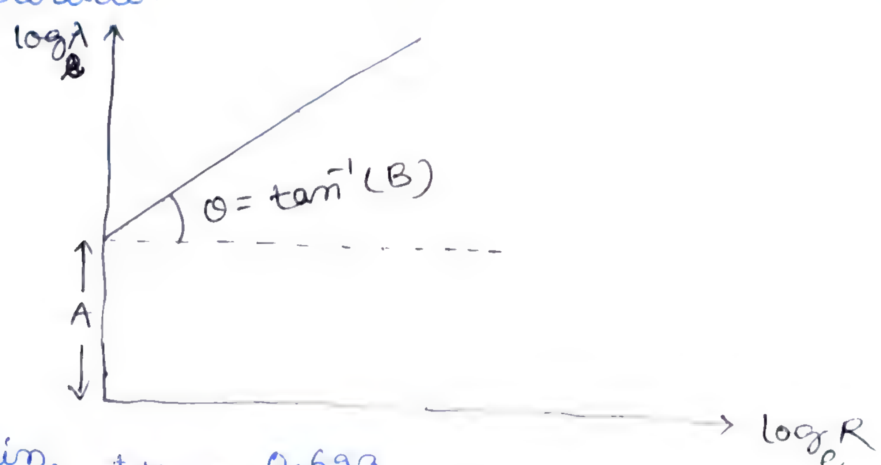
→ The distance through which α-particle travels in a specific medium before stoping to ionising, is called range of α-particle.

Geiger-Nuttal proved that, the range of α particle (R) in gaseous medium and the disintegration constant (λ) of the radio active substance emiting the α-particle are connected by a simple relation known as Geiger-Nuttal law.

Geiger Nuttal law is given by,

$$\boxed{\log_e \lambda = A + B\log_e R}$$, where A & B are const.

From Geiger Nuttal law, a greater the value of disintegration constant, a greater the range of α-particle.



Again, $t_{1/2} = \dfrac{0.693}{\lambda}$

$\lambda = \dfrac{0.693}{t_{1/2}}$

$\therefore \log_e\left(\frac{0.693}{t_{1/2}}\right) = A + B \log_e R$

or, $\log_e 0.693 - \log_e(t_{1/2}) = A + B \log_e R$

or, $-\log_e(t_{1/2}) = \underbrace{(A - \log_e 0.693)}_{\text{constant}} + B \log_e R$

or, $\boxed{-\log_e(t_{1/2}) = c + B \log_e R}$

This is the relation between range of α-particle and half life of a radio active substance from which α-particle is emited.

Q. Define straggling of the range of α-particle. Why straggling of the range of α-particle occur.

→ The α-particles of same initial kinetic energy have more or less range in a particulare medium. There is a small spread in the values ~~spread~~ ~~in the values~~ of the range about a mean value which is known as straggling of the range of α-particle.

Straggling of the range of α particle occurs mainly due to two reasons –

i) There is a fluctuation in the number of collision of α-particle with different molecules present in the medium. Thats why different α-particle with same initial kinetic energy would have different ranges in a specific medium.

ii) The kinetic energy loss per collision has also fluctuation i.e. in different collision the loss of kinetic energy is different.

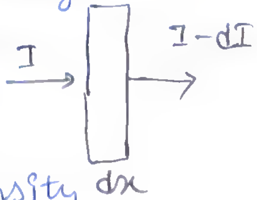
γ- Decay / Disintegration

Q. i) How does the intensity of γ- ray varies with distance when it interacts with a substance.

ii) Define half - thickness and radiation length or relaxation length of a specific medium.

→ i) γ- ray either absorbed or scattered from their path while passing through a specific material. Due to this reason the intensity of γ- ray reduced with the distance travel in a specific medium.

Let, I is the intensity of incident γ-ray on a ~~step~~ slab ~~of~~ of width dx as shown in the figure. The decrease in intensity is dI.



dI is proportional to intial intensity and width of the slab.

$$dI \propto dx$$

$$dI \propto I$$

$$\therefore \quad dI \propto -I\,dx$$

$$\text{or,}\quad dI = -\mu I\,dx,$$

$$\text{or,}\quad \frac{dI}{I} = -\mu\,dx \quad \text{——— (i)}$$

The -ve sign indicate intensity decreases with distance

μ = absorption cofficient (it depends on medium)

Integrating both with proper limit,

$$\int_{I_0}^{I} \frac{dI}{I} = -\mu \int_{0}^{x} dx$$

I_0 = initial intensity.

$$\text{or,}\quad [\ln I]_{I_0}^{I} = -\mu x$$

$$\text{or,}\quad \ln \frac{I}{I_0} = -\mu x$$

$$\text{or,}\quad \boxed{I = I_0 e^{-\mu x}}$$

So, the intensity of γ- ray decreases with distance exponentially

ii) Half length or half-thickness: The thickness of a material required to reduce the intensity of γ-ray by half of its initial intensity, is known as half-length or half thickness.

Let, $x = d_{1/2}$, $I = \frac{I_0}{2}$

$$\therefore \frac{I_0}{2} = I_0 e^{-\mu d_{1/2}}$$

or, $e^{\mu d_{1/2}} = 2$

or, $\mu d_{1/2} = \ln 2$

or, $d_{1/2} = \frac{\ln 2}{\mu}$

or, $\boxed{d_{1/2} = \frac{0.693}{\mu}}$

Radiation length: The radiation length is the thickness of the slab required to reduce the intensity of γ-ray by $1/e$ times of initial intensity of γ-ray.

Let, $x = L$, $I = I_0/e$

$$\therefore \frac{I_0}{e} = I_0 e^{-\mu L}$$

or, $e^{-1} = e^{-\mu L}$

or, $\mu L = 1$

or, $\boxed{L = \frac{1}{\mu}}$ This is the expression for radiation length or relaxation length.

Q. Initial intensity of γ-ray is $30\ W/m^2$ and absorption coefficient of the medium is 0.22, find the intensity of γ-ray at distance 10 m. Also find the value of half thickness and radiation length.

→ $I_0 = 30\ W/m^2$ $\mu = 0.22$

$I = I_0 e^{-\mu x}$

$= 30 \times e^{-0.22 \times 10}$

$= 30 \times e^{-2.2}$

$= 30 \times 0.118$

$= ~~3.36~~\ 3.324.$

half thickness

$d_{1/2} = \frac{0.693}{0.22}$

$= 3.15$

radiation length

$L = \frac{1}{\mu}$

$= \frac{1}{0.22}$

$= 4.545$

Q. Mention different processes through which γ-ray absorb by a material.
OR, Explain how γ- photon interact with matter while passing through it.

→ There are mainly three processes through which a γ-ray loss its kinetic energy. These processes are –

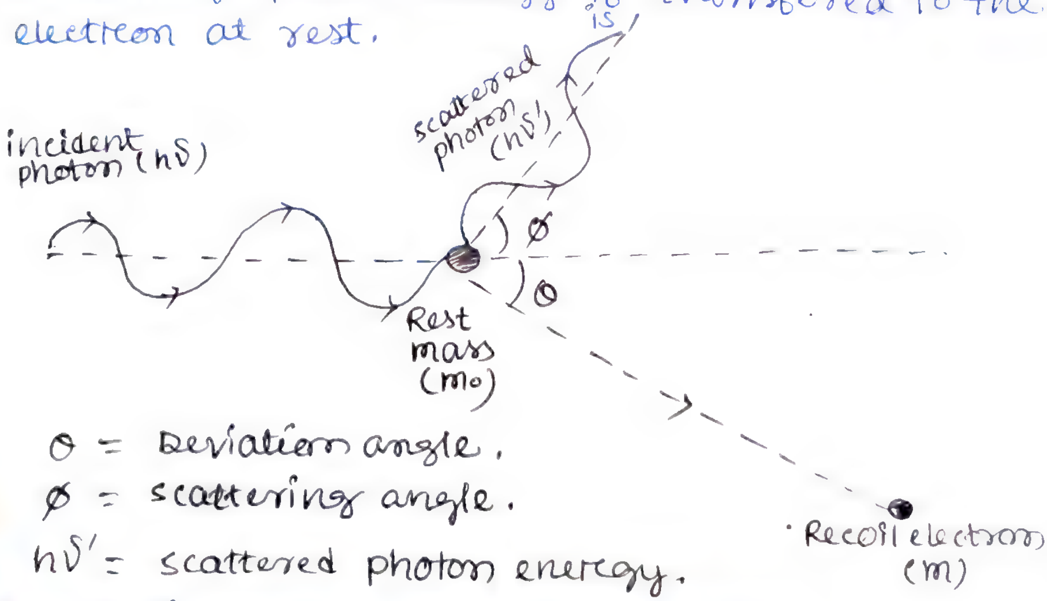
i) <u>photoelectric effect</u>: In photoelectric effect γ-photon collides with different atoms present in the medium in this process the total energy $h\nu$ of γ-photon transfered to the electron, as a result the electron is ejected from the atom.

According to Einstein photoelectric effect,

$$h\nu = \phi + \frac{1}{2} m v_{max}^2$$

ϕ = work-function
$\frac{1}{2} m v_{max}^2$ = maximum K.E of electron.

ii) <u>compton scattering</u>: In this process photon is scattered by electron at rest. In this process a portion of photon energy ~~it~~ is transfered to the electron at rest.



θ = Deviation angle.
ϕ = scattering angle.
$h\nu'$ = scattered photon energy.

The kinetic energy of recoil electron,

$$E_K = h\nu - h\nu'$$
$$= h\nu \left[\frac{\frac{h\nu}{m_0 c^2}(1-\cos\phi)}{1 + \frac{h\nu(1-\cos\phi)}{m_0 c^2}} \right]$$

iii) electron-positron pair production: ~~it~~ is causes.
In this process γ photon disappears and converted into an electron-positron pair. In this process minimum energy require is 1.02 MeV

$$\underset{\substack{1.02 \\ (\text{Min})}}{\gamma(\text{photon})} \longrightarrow \underset{0.51}{\text{electron}(e^-)} + \underset{0.51}{\text{positron}(e^+)}$$

When a γ ray with total energy equal or more than 1.02 MeV travel through the strong electric field of a nucleus the photon or γ-ray converted into electron positron pair by the mass energy equivalence relation.

$E = mc^2$

The pair production is only possible in high density medium like nucleus, its impossible in free space or vaccum.

Q. i) why minimum energy required in pair production is 1.02 MeV.
ii) why pair production is not possible in vaccum or free space.

→ i) In pair production γ-photon is converted into pair of electron and positron

$$\gamma\text{-photon} \longrightarrow \underset{(e^-)}{\text{electron}} + \underset{(e^+)}{\text{positron}}$$

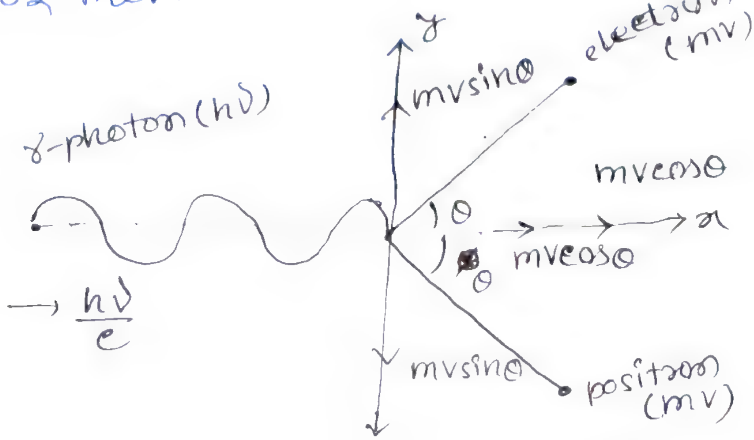
electron and positron is anti particle of each other, they have equal mass.

Rest mass energy of electron = Rest mass energy of positron
= 0.51 MeV

Minimum energy of γ-photon required for the pair production = rest mass energy of electron + rest mass " " positron.

= 0.51 MeV + 0.51 MeV.
= 1.02 MeV.

ii)



Let, the momentum of incident γ-photon is $\frac{h\nu}{c}$, the momentum of electron and positron produced by the pair production is mv.

By using momentum conservation in x axis,

momentum before pair production = momentum after pair production.

$$\frac{h\nu}{c} = mv\cos\theta + mv\cos\theta.$$

or, $h\nu = 2mvc\cos\theta$

or, $h\nu = 2mc^2 . \frac{v}{c} \cos\theta \longrightarrow (i)$ c = velocity of light.

$\frac{v}{c} < 1$; $\cos\theta \leq 1$

from equation (i)

$$h\nu < 2mc^2$$

so, energy is not conserved in this process. If mass energy and momentum is not simultaniously conserved in a process then the process is impossible in nature. Thats why pair production is not possible in free space or vaccum.

* **Note:** all questions of compton effect from 5th sem.

Q. Explain the terms – i) Internal photoelectric effect or internal convertion. (2017)
ii) electron-positron annhilation.

→ i) Internal convertion: when a nucleus passes from a higher exited state to a lower energy state, the difference in energy of the two states is emited as γ-ray or higher energy photon. sometimes its happens that the exited nucleus returns to the ground state by giving up ex excitation energy to the orbit electron. and Due to absorption of photon the electron is ejected from the atom, this phenomenon is known as internal photoelectric effect or internal convertion

ii) electron-positron annhilation: (2018)

when a positron meets with an electron the two particle destroys themselves by producing γ-ray or photon, These phenomenon is known as electron-positron annhilation.

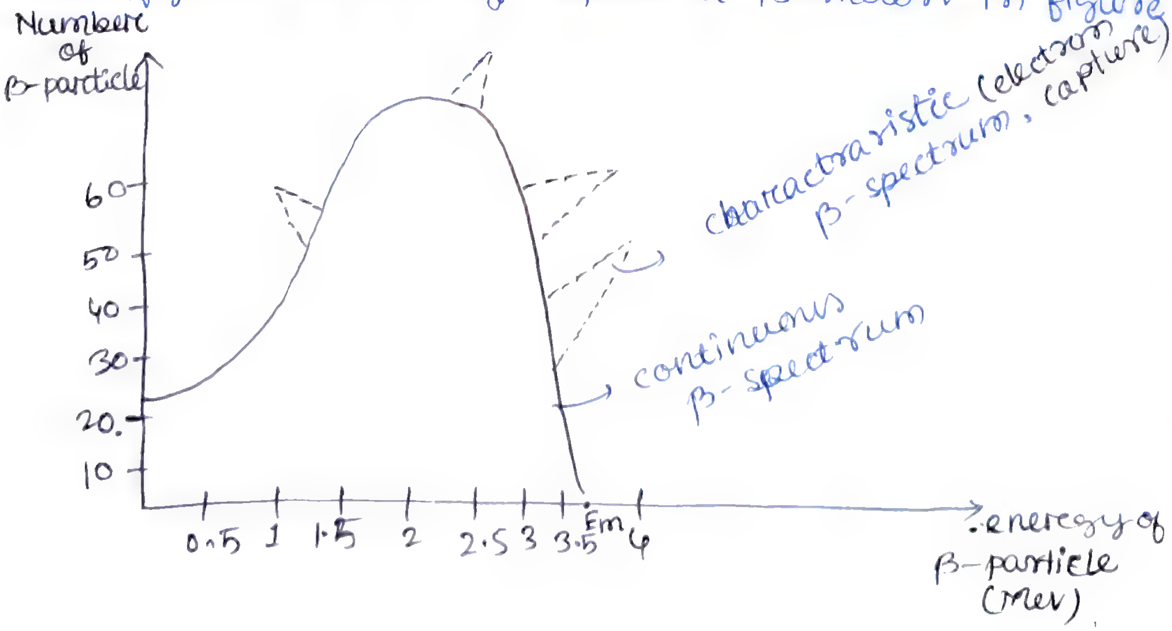
electron + positron ⟶ γ-ray
(e⁻) (e⁺) (γ-photon).

This is totally reverse process of pair of production. Each electron-positron pair with anti-parallel spin disappears giving rise to two photons of same frequency but travelling in opposite direction for the conservation of mass energy and momentum.
With parallel spin of positron-electron pair it give rise to three γ-photons of different frequency

Q. $\boxed{\beta\text{- Disintegration}}$

Q. Why β-disintegration takes place in unstable nuclei or radio active nuclei.
OR, Explain the emission of β-particle from a nuclei
→ same as N/Z graph.

Q. Describe the nature of β-ray spectrum in suitable graph. Discuss the important factors related to this spectra.
→ β-ray spectrum: The energy of different β-particles emited from a radio active source is studied with the help of β-ray spectrometer. These are based on the principle of separation of β-particle with different kinetic energy by applying strong magnetic field. The general distribution of energy in a β-ray spectra is shown in figure



Different factors related to β- spectrum are:
i) The β spectrum is continuous having kinetic energy of β particle from 0 to a certain well defined limit known as end point energy (E_m).

ii) The area under the curve is directly proportional to the no. of β-particles.

iii) There is a number of ~~sp~~ sharp lines or peaks in the β- spectrum which are found to be prominent on the photographic plate. These peaks are known as characteristics β-spectrum, These peaks indicates high energy photon emitted during electron capture.

iv) Here is a definite upper limit or end point energy for β- particles emited by the nucleus, which is different for different β-emited nuclei.

***Q. Explain theoritically how ~~the~~ neutrino hypothesis solves the conservation of momentum and mass energy of β-decay. (2017)
OR, How neutrino hypothesis explain the continuous β-spectrum. 2018
OR, Explain Pauli Neutrino hypothesis.

→ If it assumed that when a nucleus emits β-particle, a neutron in the nucleus changes to a proton and a β-particle, then all β-particle from a given radioactive substance must be emitted with the same kinetic energy. But actual measurements show that only a few β-particle ~~and~~ are emitted with a maximum value of energy. The majority of β-particle are emitted with smaller energy ranging from zero to a maximum value (end point energy Em). The law of conservation of energy and momentum donot hold good for single particle β^+ or β^- emission.

All these difficulties have been overcome by supposing the existance of another particle called neutrino and its anti-particle the anti-neutrino to be ~~am~~ emitted simultaneously along with the β-particle. Its existance was first predicted by Pauli on theoritical

grounds in 1930 and has been confirmed experimentally in 1956.

The β-particle and neutrino particle escape with a constant total energy equal to the difference between the energies of the original and the final nucleus. The continuous energy distribution arises from the variable manner in which the total energy is shared between the β-particle and the neutrino. If the difference in energy between original and final nucleus be E_m, then $E_m = E_\beta + E_\gamma$

<u>Properties of neutrino:</u>

i) The neutrino as well as the antineutrino has zero rest mass.

ii) Neutrino has no charge.

iii) It has an angular momentum or spin $= \frac{1}{2}\hbar$

iv) It interacts extremely weakly with matter.

Q. Define electron capture.

→ Both +ve electron (positron) and -ve electrons (negatron) are emitted from radioactive nuclei. This phenomenon is called β-decay. The reverse process is electron capture in which excited nucleus absorbs one of its own orbit electron. If the nucleus absorbs electron from K-th shell then it is also called K-capture.

$$\boxed{p + e^- \longrightarrow n + \nu_e \text{ (netrino)}}$$

Q. Define thermal neutrino. Write down uses of thermal neutrino.

Nuclear models

To describe different propertise nucleus.

1. liquid drop model. *✓
2. Fermi gas model.
3. Shell model. 4. single particle shell model
5. collective model 6. Deformation model.
7. Nilson model.

4, 8, 20, 50, 80, 126 → Highly stable.
↳ shell model
↳ called magic number.

Q. write down the similarity between a nucleus and a liquid drop.

→ In liquid drop model a nucleus is considered as incompresable liquid drop. The similarities between liquid drop with nucleus are given below –

i) A liquid drop is consist of large number of molecules, similarly a nucleus is made of large no. of proton and neutrons.

ii) In incompresable liquid drop density is constant similarly density of nucleus is constant (10^{17} kg/m^3)

iii) liquid drops are form due to short range london force and venderwaal force, similarly a nucleus is form due to ~~stron~~ short range nuclear force.

iv) small drops of liquid combined to form a bigger liquid drop, this is similarity to nuclear fusion.

$$4\,{}_1H^1 \longrightarrow {}_2He^4 + 2\,{}_1p^1$$

v) A big liquid drop breaks into smaller droplet due to their unstability, this is similar to nuclear fission.

$${}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{56}Ba^{144} + {}_{36}Kr^{\cancel{89}\,82} + 3\,{}_0n^1$$

Q. i) Define magic numbers.
ii) Write down the limitations of liquid drop model.

→ i) From experimental result there are certain number of proton and neutron for which a nucleus is highly stable. These numbers are known as magic numbers.
magic numbers are 2, 8, 20, 50, 82, 126
some magic no. 40 & 28.

ex. $_2He^4$, $_8O^{16}$, $_{20}Ca^{40}$ etc.

ii) Some limitations of liquid drop model are -
(i) By using liquid model we cannot able to explain magic number.
b) liquid drop model cannot explain higher excited state of a nucleus.
c) pairing of proton and neutron cannot be explain by liquid drop theory.
d) The exchange of meson (K-meson, π-meson) in a nuclear force cannot be explain by liquid drop model.

Q. i) Describe the phenomenon of nuclear fission. Explain nuclear fission with product percentage graph.
ii) Explain nuclear fission on the basis of liquid drop model.

→ i) Nuclear fission: The process of breaking up of the nucleus of a heavy atom into two more or less equal segments with the release of a large amount of energy, is known as nuclear fission.
For ex.-

$$_{92}U^{235} + {}_0n^1 \longrightarrow [{}_{92}U^{236}] \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + E$$

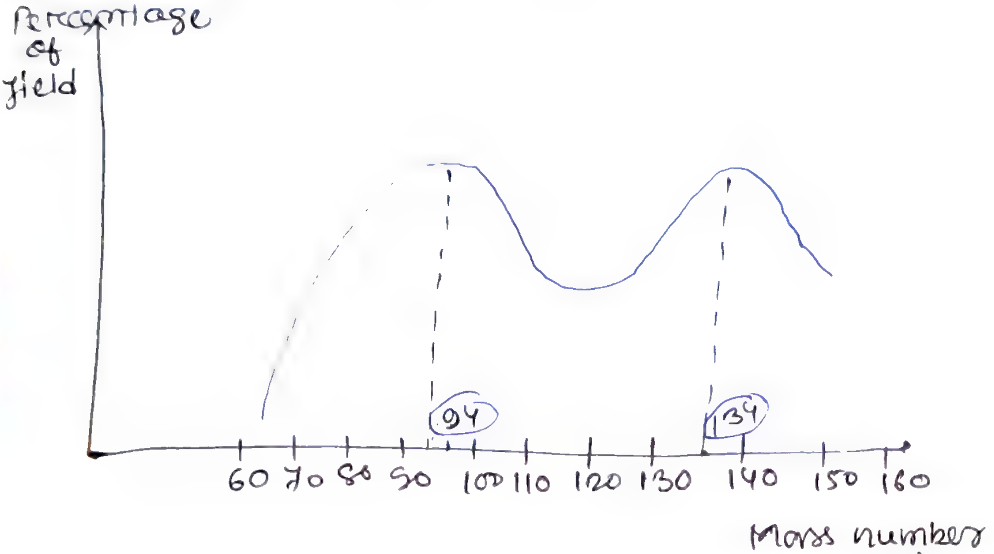
When $_{92}U^{235}$ is bombarded with thermal neutron it splits into $_{56}Ba^{141}$ and $_{36}Kr^{92}$ and 3 neutrons are emitted along with large amount of energy.

The U^{235} nuclei do not all split up into those of Ba and Kr. They may divided into the nuclei of several pairs of electrons lying in the central region of the periodic table with slightly unequal nuclear masses. These are known as fission fragments. Thus another mode of U^{235} fission

$$_{92}U^{235} + {}_0n^1 \longrightarrow [{}_{92}U^{236}] \rightarrow {}_{40}Zr^{98} + {}_{52}Te^{135} + 3\,{}_0n^1 + E$$

E = energy

If we plot the percentage of yields as function of the atomic number A, we get a curve of the type ~~of~~ as shown in fig.



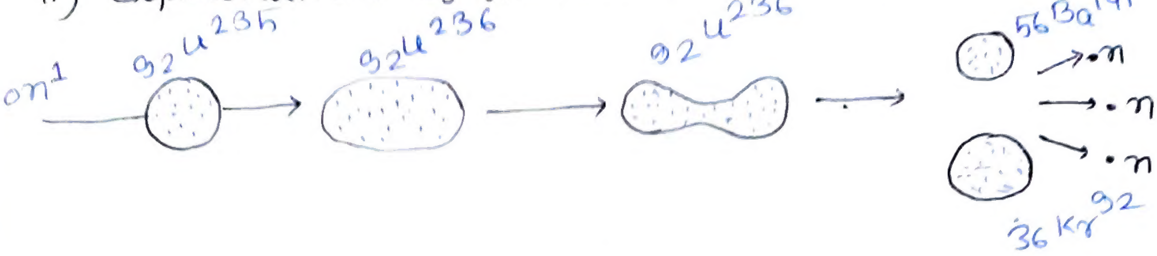
The yield is maximum at A = 95 and 139 and the distribution is called asymmetric

The yield is minimum at A = 118 which corresponds to symmetric spliting of U^{235} nucleus.

Fission may occurs spontaneously...

or it may be induced. Most of the heavier isotops of the elements with $Z > 82$ shows spontaneous fission. In such a case the no. of protons in the nucleus is very large so that the electrostatic force of repulsion between them exceeds the nuclear binding force.

ii) Explanation of fission on liquid drop model:



Bohr and wheeler explained the phenomenon of nuclear fission on the liquid drop model of the nucleus. The fissile nucleus is normally maintained in equilibrium under the combined action of short range nuclear forces of attraction among the nucleons in it which try to maintain spherical shape of the nucleus as such and the columb forces of repulsion among the protons in it which try to distort its shape. when some energy is imparted to the drop, say through the capture of a neutron, oscillation are setup in the drop which tends to distort the spherical shape of the nucleus, while the surface tension forces try to restore it. When the excitation energy is sufficiently large, the compound nucleus is an excited state and is sufficiently distorted in shape like that of a dumb-bell. when the columb force of repulsion between the two halves of this dumbell exceeds the nuclear forces holding the nucleons, the nucleus breaks upto two fragments and fission is said to take place. The various steps from neutron capture to fission of U^{235} nucleus are show in figure.

Q. Explain concept of compound nucleus with example.

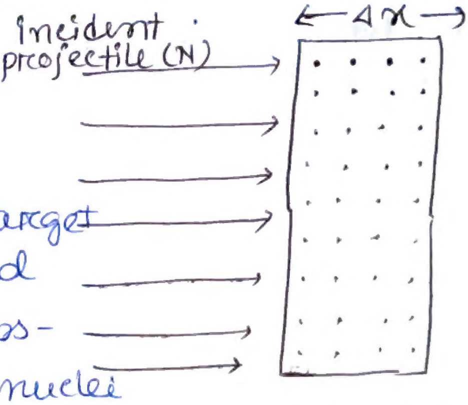
OR, Give the mechanism of nuclear reaction with the help of liquid drop model.

Q. i) Define nuclear cross section. Find an expression for nuclear reaction cross section, write down its unit.

ii) Writedown the geometrical significance of cross section of nuclear reaction.

→ i) An important parameter a nuclear reaction is the reaction cross section (σ), reaction cross section is a quantative measurement of the probability of occurance of a nuclear reaction.

Let, a parallel beam of N no. of mono-energetic projectiles per unit time incident normally on a target foil of surface area A and thickness Δx, the target subs-tance has n number of nuclei per unit volume.



Target foil

Let, ΔN number of nuclei in the foil undergoes nuclear reaction in this process, ΔN will be pro-portional to — i) the intensity of projectile particle.

ii) The no. of target nuclei presents in the foil.

$\Delta N \propto I$, $I = \frac{N}{A}$

$\Delta N \propto n A \Delta x$

$$\Delta N \propto \frac{N}{A} \cdot n A \Delta x$$

or, $\Delta N \propto n N \Delta x$

or, $\Delta N = \sigma n N \Delta x$, σ = proportional constant.

or, $\sigma = \frac{\Delta N}{n N \Delta x}$ ——→ (i)

Dimension of $\sigma \rightarrow [L^2]$

'σ' has a dimension same as surface area, that's why the proportional constant 'σ' is known as nuclear reaction cross section.

From eqn. (i)

$\sigma = \frac{\Delta N}{n_A N}$, n_A = no. of pertarget nuclei per unit surface area.

$n = \frac{N_0}{A \cdot \Delta x}$

$n \Delta x = \frac{N_0}{A}$

= per unit surface area.

'σ' is the propability of occurance of a nuclear reaction when a single particle ($\Delta N = 1$) falls on a target sample with one target nuclei per unit area.

The unit of nuclear cross section is Barn.

$$\boxed{1 \text{ Barn} = 10^{-28} m^2}$$

ii) Geometrically the reaction cross section area is the amount of area of cross section of an imaginary dice for each target nuclei such that ~~its~~ If the incident projectile passes through it the reaction occurs.

Geometrically $\sigma = \pi R^2$, where R = Radius of target nuclei.

unit-II (Quantum mechanics)

1. What is blackbody and what do you mean by blackbody radiation.

→ Blackbody: A perfectly blackbody is one which absorbs all the heat radiations corresponding to all wavelengths incident on it.

When radiation falls on matter, a part of it may be reflected, a part may transmitted and a part may be absorbed.

i.e $r + t + a = 1$,

a = absorbed part of incident radiation.
t = transmitted ...
r = reflected ...

If for any body $r = 0$, $t = 0$ then $a = 1$ such a body is called perfectly blackbody.

A blackbody behaves as a perfect ~~radiation~~ radiator. When heated radiation emitted by a blackbody is knowns as blackbody radiation.

Q2. Discuss the characteristics of or the general nature of the spectrum of blackbody radiation.

→ <u>Nature of blackbody radiation spectrum:</u>

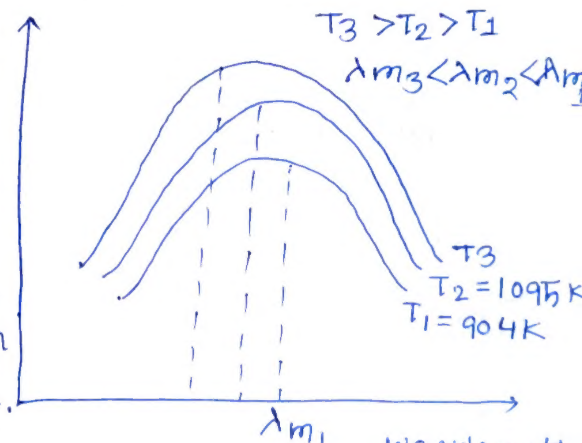


Fig: Experimental curve of black body radiation.

a) Again at a given temp., the energy is not uniformly distributed in the radiation spectrum of a blackbody.

b) At a given temperature, the intensity of heat radiation increases with increase in wavelength and at a particular wavelength (λm) its value is maximum. With ~~future~~ further increase in wavelength the intensity of heat radiation decreases.